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Resummed jet rates for heavy quark production in e^+e^- annihilation[*]

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Abstract. Expressions for Sudakov form factors for heavy quarks are presented. They are used to construct resummed jet rates in e^+e^- annihilation. Predictions are given for production of bottom quarks at LEP and top quarks at the Linear Collider.

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1 Introduction

The formation of jets is the most prominent feature of perturbative QCD in e^+e^- annihilation into hadrons. Jets can be visualized as large portions of hadronic energy or, equivalently, as a set of hadrons confined to an angular region in the detector. In the past, this qualitative definition was replaced by quantitatively precise schemes to define and measure jets, such as the cone algorithms of the Weinberg–Sterman [1] type or clustering algorithms, e.g. the Jade [2,3] or the Durham scheme ($k_\perp$ scheme) [4]. A refinement of the latter one is provided by the Cambridge algorithm [5]. Equipped with a precise jet definition the determination of jet production cross sections and their intrinsic properties is one of the traditional tools to investigate the structure of the strong interaction and to deduce its fundamental parameters. In the past decade, precision measurements, especially in e^+e^- annihilation, have established both the gauge group structure underlying QCD and the running of its coupling constant α_s over a wide range of scales. In a similar way, also the quark masses should vary with the scale.

A typical strategy to determine the mass of, say, the bottom-quark at the centre-of-mass (c.m.) energy of the collider is to compare the ratio of three-jet production cross sections for heavy and light quarks [6,7,8,9,10]. At jet resolution scales below the mass of the quark, i.e. for gluons emitted by the quark with a relative transverse momentum $k_\perp$ smaller than the mass, the collinear divergences are regularized by the quark mass. In this region mass effects are enhanced by large logarithms $\ln(m_b/k_\perp)$,

increasing the significance of the measurement. Indeed, this leads to a multiscale problem since in this kinematical region also large logarithms $\ln(\sqrt{s}/k_\perp)$ appear such that both logarithms need to be resummed simultaneously. A solution to a somewhat similar two-scale problem, namely for the average sub-jet multiplicities in two- and three-jet events in e^+e^- annihilation was given in [11]. We report here on the resummation of such logarithms in the $k_\perp$-like jet algorithms [12] and provide some predictions for heavy quark production. A preliminary comparison with next-to-leading order calculations of the three-jet rate [6,13,14,15] is presented.

2 Jet rates for heavy quarks

A clustering according to the relative transverse momenta has a number of properties that minimize the effect of hadronization corrections and allow an exponentiation of leading (LL) and next-to-leading logarithms (NLL) [4] stemming from soft and collinear emission of secondary partons. Jet rates in $k_\perp$ algorithms can be expressed, up to NLL accuracy, via integrated splitting functions and Sudakov form factors [4]. For a better description of the jet properties, however, the matching with fixed order calculations is mandatory. Such a matching procedure was first defined for event shapes in [16]. Later applications include the matching of fixed-order and resummed expressions for the four-jet rate in e^+e^- annihilation into massless quarks [17,18]. A similar scheme for the matching of tree-level matrix elements with resummed expressions in the framework of Monte Carlo event generators for e^+e^- processes was suggested in [19] and extended to general collision types in [20].

We shall recall here the results obtained in [12] for heavy quark production in e^+e^- annihilation. In the quasi-collinear limit [21,22], the squared amplitude at tree-level

fulfils a factorization formula, where the splitting functions P_{ab} for the branching processes $a \to b + c$, with at least one of the partons being a heavy quark, are given in $D = 4 - 2\epsilon$ dimensions by

$$P_{QQ}(z,q) = C_F \left[\frac{1+z^2}{1-z} - \epsilon (1-z) - \frac{2z(1-z)m^2}{q^2 + (1-z)^2 m^2} \right] ,$$

$$P_{gQ}(z,q) = T_R \left[1 - \frac{2z(1-z)}{1-\epsilon} + \frac{2z(1-z)m^2}{(1-\epsilon)(q^2 + m^2)} \right] , \quad (1)$$

where z is the usual energy fraction of the branching, and q^2 is the space-like transverse momentum. As expected, these splitting functions match the massless splitting functions in the limit $m \to 0$ for q^2 fixed. Furthermore, for $q \ll (1-z)\, m$ the splitting function P_{QQ} is not any more enhanced at $z \to 1$, which is the known "dead cone" [23] effect. The splitting function

$$P_{gg}(z) = C_A \left[\frac{z}{1-z} + \frac{1-z}{z} + z(1-z) \right] . \quad (2)$$

obviously does not get mass corrections at the lowest order.

Branching probabilities are defined through [12]

$$\Gamma_Q(Q,q,m) = \int\limits_{q/Q}^{1-q/Q} dz \frac{q^2}{q^2 + (1-z)^2 m^2} P_{QQ}(z,q)$$

$$= \Gamma_Q(Q,q,m=0) + C_F \left[\frac{1}{2} - \frac{q}{m} \arctan\left(\frac{m}{q}\right) \right.$$

$$\left. - \frac{2m^2 - q^2}{2m^2} \ln\left(\frac{m^2 + q^2}{q^2}\right) \right] ,$$

$$\Gamma_f(Q,q,m) = \int\limits_{q/Q}^{1-q/Q} dz \frac{q^2}{q^2 + m^2} P_{gQ}(z,q)$$

$$= T_R \frac{q^2}{q^2 + m^2} \left[1 - \frac{1}{3}\frac{q^2}{q^2 + m^2} \right] ,$$

$$\Gamma_g(Q,q) = \int\limits_{q/Q}^{1-q/Q} dz\, P_{gg}(z) = 2C_A \left(\ln\frac{Q}{q} - \frac{11}{12} \right) , (3)$$

and the Sudakov form factors, which yield the probability for a parton experiencing no emission of a secondary parton between transverse momentum scales Q down to Q_0, read

$$\Delta_Q(Q,Q_0) = \exp\left[-\int\limits_{Q_0}^{Q} \frac{dq}{q} \frac{\alpha_s(q)}{\pi} \Gamma_Q(Q,q) \right] ,$$

$$\Delta_g(Q,Q_0) = \exp\left[-\int\limits_{Q_0}^{Q} \frac{dq}{q} \frac{\alpha_s(q)}{\pi} (\Gamma_g(Q,q) + \Gamma_f(Q,q)) \right] ,$$

$$\Delta_f(Q,Q_0) = [\Delta_Q(Q,Q_0)]^2 / \Delta_g(Q,Q_0) , \quad (4)$$

where $\Gamma_f(Q,q)$ accounts for the number $n_f^{(l,h)}$ of active light or heavy quarks. Jet rates in the $k_\perp$ schemes can be expressed by the former branching probabilities and Sudakov form factors. For the two- and the three-jet rates

$$\mathcal{R}_2 = [\Delta_Q(Q,Q_0)]^2 , \quad (5)$$

$$\mathcal{R}_3 = 2 [\Delta_Q(Q,Q_0)]^2 \int\limits_{Q_0}^{Q} \frac{dq}{q} \frac{\alpha_s(q)}{\pi} \Gamma_Q(Q,q) \Delta_g(q,Q_0) ,$$

where Q is the c.m. energy of the colliding e^+e^-, and $Q_0^2 = y_{\text{cut}} Q^2$ plays the role of the jet resolution scale. Single-flavour jet rates in Eq. (5) are defined from the flavour of the primary vertex.

In order to catch which kind of logarithmic corrections are resummed with these expressions it is illustrative to study the above formulae in the kinematical regime such that $Q \gg m \gg Q_0$. Expanding in powers of α_s, jet rates can formally be expressed as

$$\mathcal{R}_n = \delta_{n2} + \sum_{k=n-2}^{\infty} \left(\frac{\alpha_s(Q)}{\pi} \right)^k \sum_{l=0}^{2k} c_{kl}^{(n)} , \quad (6)$$

where the coefficients $c_{kl}^{(n)}$ are polynomials of order l in $L_y = \ln(1/y_{\text{cut}})$ and $L_m = \ln(m^2/Q_0^2)$. The coefficients for the first order in α_s are given by

$$c_{12}^{(2)} = -c_{12}^{(3)} = -\frac{1}{2}C_F(L_y^2 - L_m^2) ,$$

$$c_{11}^{(2)} = -c_{11}^{(3)} = \frac{3}{2}C_F L_y + \frac{1}{2}C_F L_m . \quad (7)$$

The coefficients for the second order in α_s can be found in [12], as well as the corresponding result for the four-jet rate.

The impact of mass effects can be highlighted by two examples, namely by the effect of the bottom quark mass in e^+e^- annihilation at the Z-pole (Fig. 1, up), and by the effect of the top quark mass at a potential Linear Collider operating in the TeV region (Fig. 1, down). In Fig. 1, leading order (LO) and next-to-leading order (NLO) predictions for three-jet rates are compared with the NLL result as obtained by numerical integration from Eq. (5). For a comparison with the dead cone approximation see also [12]. While in the case of bottom quarks at LEP1 energies the overall effect of the quark mass is at the few-percent level, this effect becomes tremendous for top quarks at the Linear Collider. Fixed order predictions for b-quark production clearly fail at very low values of y_{cut}, by giving unphysical values for the jet rate, while the NLL predictions keep physical and have the correct shape. The latter is an indication of the necessity for performing such kind of resummations. Fixed order predictions work well for top production at the Linear Collider, a consequence of the strong cancellation of leading logarithmic corrections, and are fully compatible with our NLL result.

Matching between fixed-order calculations and resummed results is in progress [24].
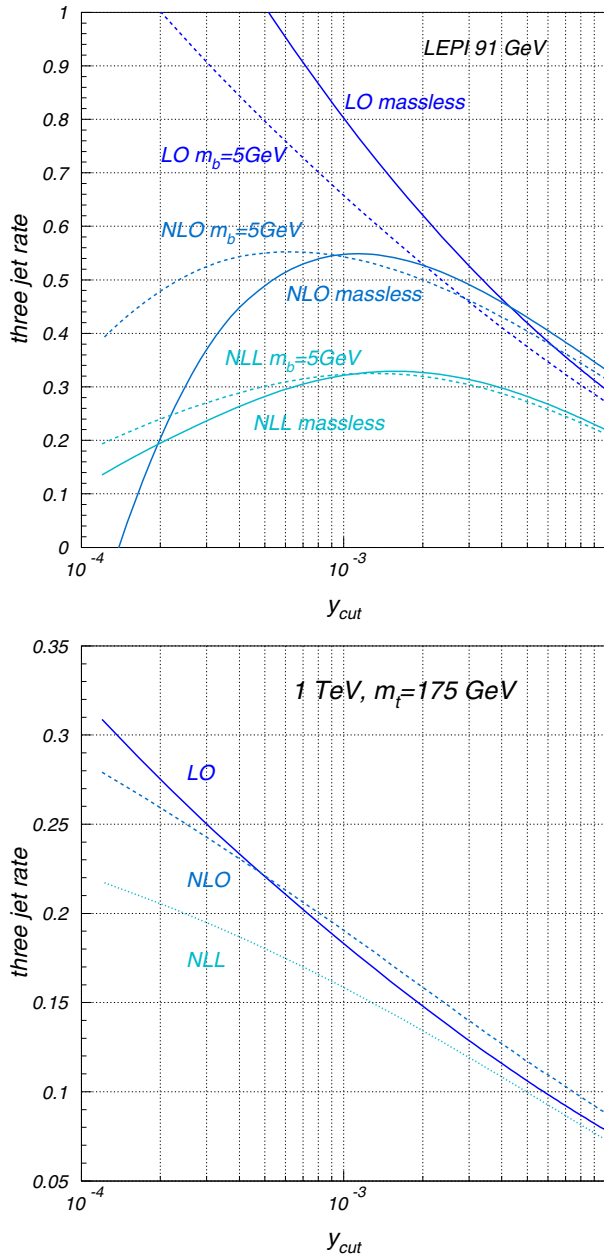
Fig. 1. Effect of a *b*-mass of 5 GeV on the single-flavour three-jet rate at LEP1 energies as a function of the jet resolution parameter in the Cambridge algorithm (upper plot). Effect of a *t*-mass of 175 GeV on the single-flavour three-jet rate as a function of the jet resolution parameter, at a potential Linear Collider operating at c.m. energies of 1 TeV (lower plot). Predictions are provided at LO, NLO and NLL accuracy

3 Conclusions

Sudakov form factors involving heavy quarks have been employed to estimate the size of mass effects in jet rates in e^+e^- annihilation into hadrons. These effects are sizeable and therefore observable in the experimentally relevant region. A preliminary comparison with fixed order results have been presented, and showed good agreement.

References

1. G. Sterman, S. Weinberg: Phys. Rev. Lett. **39**, 1436 (1977)
2. W. Bartel et al. [JADE Collaboration]: Z. Phys. C **33**, 23 (1986)
3. S. Bethke et al. [JADE Collaboration]: Phys. Lett. B **213**, 235 (1988)
4. S. Catani, Y.L. Dokshitzer, M. Olsson, G. Turnock, B.R. Webber: Phys. Lett. B **269**, 432 (1991)
5. Y.L. Dokshitzer, G.D. Leder, S. Moretti, B.R. Webber: JHEP **9708**, 001 (1997) [hep-ph/9707323]
6. G. Rodrigo, A. Santamaria, M.S. Bilenky: Phys. Rev. Lett. **79**, 193 (1997) [hep-ph/9703358]
7. P. Abreu et al. [DELPHI Collaboration]: Phys. Lett. B **418**, 430 (1998)
8. A. Brandenburg, P.N. Burrows, D. Muller, N. Oishi, P. Uwer: Phys. Lett. B **468**, 168 (1999) [hep-ph/9905495]
9. R. Barate et al. [ALEPH Collaboration]: Eur. Phys. J. C **18**, 1 (2000) [hep-ex/0008013]
10. G. Abbiendi et al. [OPAL Collaboration]: Eur. Phys. J. C **21**, 411 (2001) [hep-ex/0105046]
11. S. Catani, B.R. Webber, Y.L. Dokshitzer, and F. Fiorani: Nucl. Phys. B **383**, 419 (1992)
12. F. Krauss, G. Rodrigo: hep-ph/0303038; hep-ph/0309326
13. G. Rodrigo: Nucl. Phys. Proc. Suppl. **54A**, 60 (1997) [hep-ph/9609213]
14. M.S. Bilenky, S. Cabrera, J. Fuster, S. Marti, G. Rodrigo, A. Santamaria: Phys. Rev. D **60**, 114006 (1999) [hep-ph/9807489]
15. G. Rodrigo, M.S. Bilenky, A. Santamaria: Nucl. Phys. B **554**, 257 (1999) [hep-ph/9905276]; Nucl. Phys. Proc. Suppl. **64**, 380 (1998) [hep-ph/9709313]
16. S. Catani, L. Trentadue, G. Turnock, B.R. Webber: Nucl. Phys. B **407**, 3 (1993)
17. L.J. Dixon, A. Signer: Phys. Rev. D **56**, 4031 (1997) [hep-ph/9706285]
18. Z. Nagy, Z. Trocsanyi: Phys. Rev. D **59**, 014020 (1999) [Erratum-ibid. D **62**, 099902 (2000)] [hep-ph/9806317]
19. S. Catani, F. Krauss, R. Kuhn, B.R. Webber: JHEP **0111**, 063 (2001) [hep-ph/0109231]
20. F. Krauss: JHEP **0208**, 015 (2002) [hep-ph/0205283]
21. S. Catani, S. Dittmaier, Z. Trocsanyi: Phys. Lett. B **500**, 149 (2001) [hep-ph/0011222]
22. S. Catani, S. Dittmaier, M.H. Seymour, Z. Trocsanyi: Nucl. Phys. B **627**, 189 (2002) [hep-ph/0201036]
23. Y.L. Dokshitzer, V.A. Khoze, S.I. Troian: J. Phys. G **17**, 1602 (1991)
24. F. Krauss, G. Rodrigo, A. Schälicke, P. Tortosa: in preparation